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RESEARCH ON THE WEIGHT RESTRICTION OF DATA ENVELOPMENT ANALYSIS - TWO-STAGE OF PAIRWISE WEIGHT RATIO INCORPORATING THE EXPERT WEIGHTING METHOD

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Abstract

The traditional DEA model generally has an unreasonable problem of "variable weight is 0," and this problem will seriously affect the discrimination of the DEA efficiency evaluation; corresponding to this shortcoming, we have also proposed a model that can solve the problem. This model is called EWM_AR (Bao, 2019). Nonetheless, in the follow-up study of "How to Expand the Application Scope of the EWM_AR Model," we found that it is indeed easy to get the lower limit of negative values when dealing with the related issues of "Interval Estimation of Pairwise Weight Ratio". Therefore, this article is re-presenting a model with a non-negative lower limit characteristic. This model is called "Two-Stage Weighted Proportional Interval Estimation"- *m*EWM_AR Model.

The research results of this article found that after implementing the "Normalization" and incorporating "the Importance of the Correlation Between the Variables" method, there are two main results produced, including: (1) It can indeed solve the unreasonable problem that the traditional DEA model generally has "variable weight is 0"; (2) It can indeed make the "Interval Estimation of Pairwise Weight Ratio" have the characteristic of non-negative lower limit. This means that the *m*EWM_AR Model will provide a more objective and a more unbiased efficiency evaluation mode.

Keywords: Data Envelopment Analysis; Expert Weighted Method; EWM_AR Model

Introduction

In the new era of rapid advances in science and technology and the Internet-Working Market, in order to stabilize and even expand the market operation base, corporate organizations can find the best strategy by evaluating their performance. Therefore, there are more and more cases where DEA is applied in various industries. In theory, the DEA efficiency assessment will naturally continue to develop towards the principles of objectivity and fairness. The point is that among the elements of natural development, in addition to having reasonable weights restriction, expanding the scope of application of the DEA model will also be another focus.

Data Envelopment Analysis (DEA) 's effectiveness evaluation mode is to use mathematical linear programming to find a set of weights for Decision Making Units (DMU*s*) that can maximize its efficiency value. Therefore, the objectivity of weights becomes an important key for efficiency evaluation. In the traditional model of DEA, there is generally an unreasonable phenomenon of "variable weight is 0." That is, when a certain variable is optimal relative to other DMUs, it may appear that only this variable is given a weight value, and other variables are 0 (Doyle & Green, 1994).

At the same time, in order to avoid the process of the "Interval Estimation of Pairwise Weight Ratio," it is easy to obtain the lower limit of negative values. Therefore, we propose a model with the characteristics of a non-negative lower limit. This model is called "Two-Stage Weighted Proportional Interval Estimation"- *m*EWM_AR Model, which can solve the two unreasonable problems mentioned above at the same time.

This article is divided into five sections: The second section illustrates the impact of "Normalization" on the "Interval Estimation of Pairwise Weight Ratio" with literature case. The third section introduces the *m*EWM_AR Model research method. Section IV case proof and discussion. Section V. Conclusion.

Take the Literature Case to Illustrate the Effect of Normalization on Interval Estimation of Pairwise Weight Ratio

In order to illustrate the critical impact of Normalization on the Interval Estimation of Pairwise Weight Ratio, this article extracted two literature case data sets as illustrative cases. Two authors of the literature include: Mazinani *et al*.(2011), Liu *et al*.(2017)

Moreover, the two literature cases are applied with the Common Weight Method (CW) and the DEA-CCR Model, respectively, to obtain the weight values (v_i, u_r) of each variable (See Appendix A); the results are shown in "Table 2.1-*a* & Table 2.1-*b*" and "Table 2.2-*a* & Table 2.2-*b*." Then, the research method of the EWM_AR Model is applied to obtain the upper and lower limits of the "Interval Estimation of Pairwise Weight Ratio"; the results are shown in "Table 2.1-*d* and Table 2.2-*d*."

- The Literature 1 of Normalization has not yet been performed: (Mazinani, 2011)

The EWM_AR Model (Bao *et al*., 2019) mentioned earlier, its inequality of the "Interval Estimation of Pairwise Weight Ratio" is as follows:

Table 2.1-*a* Weight values of each variable obtained by CW: (v_i, u_r)

v_1	v_2	u_1	u_2	u_3	u_4
0. 00000261	0	0.02073205	0	0.09268793	0.00123136

Table 2.1-*b* Weight values of each variable obtained by CCR: (v_i, u_r)

DMU	v_1	v_2	u_1	u_2	u_3	u_4
1	4.9238E-05	0	5.5171E-01	0	0	2.3485E-02
2	3.1067E-05	6.7846E-05	6.8562E-01	0	0	2.1956E-02
3	4.4086E-05	2.0008E-05	4.0494E-01	0	2.479973	2.1733E-02
4	2.7809E-05	9.9400E-05	8.6781E-01	0	0	2.2176E-02
5	5.0003E-05	0	0	11.10916	0	1.9290E-03
6	3.1643E-05	8.2178E-05	7.4063E-01	0.2834686	0	2.3391E-02
7	5.0557E-05	0	0	0	9.779715	6.8263E-03
8	3.1463E-05	6.8712E-05	6.9437E-01	0	0	2.2236E-02
9	5.0998E-05	0	0	0	4.744458	2.2319E-02
10	0	1.6453E-04	1.509662	0	0	0
11	4.9187E-05	0	0	9.938809	0	5.0090E-03
12	4.9615E-05	0	5.6972E-01	0	9.445817	0
13	5.0912E-05	0	7.2051E-02	0	0	3.0776E-02
14	4.3677E-05	2.1841E-05	4.6142E-01	0.7390980	0	2.4160E-02
15	4.8338E-05	0	6.8409E-02	0	0	2.9220E-02
16	4.8124E-05	0	0	0	0	2.9762E-02
17	4.8470E-05	9.6395E-06	0	0	4.220921	2.3353E-02
18	5.0369E-05	0	5.6439E-01	0	0	2.4025E-02
19	4.9128E-05	0	3.6527E-02	9.173694	0	6.6150E-03

Table 2.1-*c* Through *MS-Excel* data analysis, the Mean and Standard Deviation obtained

Mean	$\overline{x}_1$=7.021E+10	$\overline{x}_2$ =4.925E+09	$\overline{y}_1$ =25.217	$\overline{y}_2$ =0.308	$\overline{y}_3$=0.369	$\overline{y}_4$ =1.390E+05
Standard Deviation	S_1= 1.083E+11	S_2=7.566E+09	S_3=38.939	S_4=0.475	S_5=0.570	S_6=2.146E+05

Table 2.1-*d* Upper and lower limits of the "Interval Estimation of Pairwise Weight Ratio" in EWM_AR Model

-9.182	≤	v1/v2	≤	-22.231
-0.045	≤	v2/v1	≤	-0.109
-52.856	≤	u1/u2	≤	-127.157
-0.537	≤	u2/u3	≤	-1.292
-1.71E-06	≤	u3/u4	≤	-4.11E-06
-3.56E+03	≤	u4/u1	≤	-8.56E+03

$$(\frac{v_i^L}{v_h^U})v_h \le v_i \le (\frac{v_i^U}{v_h^L})v_h ;$$

$$(\frac{u_r^L}{u_k^U})u_k \le u_r \le (\frac{u_r^U}{u_k^L})u_k \qquad (2\text{-}1)$$

Among them, $i = 1, 2, \dots, m$, m is the number of input variables; $r = 1, 2, \dots, s$, s is the number of output variables. $\bar{x}_i$ = mean of input variables, S_i = standard deviation of input variables, $\bar{y}_r$ = mean of output variables, and S_r = standard deviation of output variables. $h = i + 1$; and when $h > m$, then $h = 1$; $v_i^U = \bar{x}_i + 3S_i$ and $v_i^L = \bar{x}_i - 3S_i$ are the upper and lower limits of v_i, respectively. $k = r + 1$; and when $k > s$, then $k = 1$; $u_r^U = \bar{y}_r + 3S_r$ and $u_r^L = \bar{y}_r - 3S_r$ are the upper and lower limits of u_r, respectively.

- The Literature 2 of Normalization has not yet been performed: (Liu, 2017)

Looking at the results of the above 8 tables (Table 2.1-a ~ Table 2.2-d), we can understand the actual unreasonable problems of the traditional DEA model, including: (1) Universally has "variable weight is 0"; (2) Because there are differences in calculation units between variables, it is easy to produce the lower limit of negative value of the "Interval Estimation of Pairwise Weight Ratio" without the normalization conversion.

Research Methods of mEWM-AR Model

We will propose a DEA mode with a non-negative lower bound. This mode is called *m*EWM_AR mode, which can solve the two unreasonable problems mentioned above.

The research method of this model is divided into two steps, and the following is its introduction:

Step 1: First, perform a linear "Normalization" to convert into a dimensionless scoring data table. And find the Mean and Standard Deviation.

"Normalization" will allow the DMU's scoring data to be re-transformed into a new data table in the range of 0~1, and the unit difference of different measures between variables will no longer exist. The normalization formula used in this study is shown in the following formula (3-1):

Assuming that the DMU has m input variables and s output variables, let

$x_i' = x_i / max(x_i) - min(x_i / max(x_i))/10$;

$y_r' = y_r / max(y_r) - min(y_r / max(y_r))/10$

(3-1)

where $i = 1, 2, \dots, m$ and $r = 1, 2, \dots, s$; $max(x_i)$ = the maximum value of the input variable, and $max(y_r)$ = the maximum value of the output variable.

After Normalization, two actions were performed on the DMU score data table that was re-obtained, including: (1) Assimilate into the theory of "the Importance of the Correlation Between the Variables" with the Expert Weight Method (EWM); (2) Implementing *MS -Excel* Data / Narrative Statistics. When

Table 2.2-*a* Weight values of each variable obtained by CW: (v_i, u_r)

v_1	v_2	v_3	u_1	u_2
0.5039527E-05	0	0.5465468E-05	0.9867493E-06	0.1183468E-04

Table 2.2-*b* Weight values of each variable obtained by CCR: (v_i, u_r)

DMU	v_1	v_2	v_3	u_1	u_2
1	1.3431E-04	0	1.4567E-04	2.6299E-05	3.1542E-04
2	1.6964E-04	0	0	0	6.2897E-04
3	2.7771E-06	7.1929E-05	3.9163E-05	7.6379E-06	0
4	5.8653E-05	0	6.3610E-05	1.1484E-05	1.3774E-04
5	1.4357E-04	5.3632E-05	1.9803E-04	3.2556E-05	4.5135E-04
6	5.3678E-06	0	2.7432E-04	1.0279E-05	0
7	2.0065E-04	2.2096E-05	0	2.2455E-05	5.1959E-04
8	7.7860E-05	8.5739E-06	0	8.7131E-06	2.0162E-04
9	7.2820E-05	1.5574E-04	0	1.1120E-05	5.0396E-04
10	6.9275E-05	3.2392E-04	0	0	1.0288E-03
11	0	1.0422E-04	4.7146E-05	0	2.9429E-04
12	9.1013E-05	1.9294E-04	0	1.3883E-05	6.2597E-04
13	3.1057E-05	1.1602E-05	4.2838E-05	7.0426E-06	9.7637E-05
14	3.8415E-05	0	3.0015E-05	8.1614E-06	0

Table 2.2-*c* Through *MS-Excel* data analysis, the Mean and Standard Deviation obtained

Mean	$\bar{x}_1$=18.308	$\bar{x}_2$=4.354	$\bar{x}_3$=1.991	$\bar{y}_1$=257.214	$\bar{y}_2$=0.238
Standard Deviation	S_1=33.237	S_2=7.879	S_3=3.583	S_4=492.729	S_5=0.429

Table 2.2-*d* Upper and lower limits of the "Interval Estimation of Pairwise Weight Ratio" in EWM_AR Model

-2.908	≤	v1/v2	≤	-6.120
-1.514	≤	v2/v3	≤	-0.344
-0.074	≤	v3/v1	≤	-0.157
-800.638	≤	u1/u2	≤	-1654.338
-0.001	≤	u2/u1	≤	-0.001

these two actions are completed, the mean ($\overline{x}'_m$, $\overline{y}'_s$) and standard deviation S_t will be obtained. (where t =1, 2,…, $(m+s)$)

Step 2: Apply the "Mean ($\overline{x}'_m$, $\overline{y}'_s$) and Standard Deviation S_t" to generate the inequality of weight restriction of the Interval Estimation of Pairwise Weight Ratio.

Combining the concepts of "AR model (Thompson, 1986) and the Law of Large Numbers," and then applying the Mean and Standard Deviation, we can generate the inequality of weight restriction of the "Interval Estimation of Pairwise Weight Ratio" re-revised in this research.

Suppose that DMU$_j$ (j =1,…, n) uses the input x_{ij} (i =1,…, m) to generate the output y_{rj} (r =1,..., s). So let the upper and lower limits of the inequality of the weight restriction of the "Interval Estimation of Pairwise Weight Ratio" be:

$$\frac{\bar{x}_h}{\bar{x}_i+3S_{\bar{x}_i}} \le \frac{v_h}{v_i} \le \frac{\bar{x}_h+3S_{\bar{x}_h}}{\bar{x}_i};$$

$$\frac{\bar{y}_k}{\bar{y}_r+3S_{\bar{y}_r}} \le \frac{u_k}{u_r} \le \frac{\bar{y}_k+3S_{\bar{y}_k}}{\bar{y}_r} \qquad (3\text{-}2)$$

Among them, i = 1, 2,…, m, h = i+1, and when $h > m$, then h =1.
r = 1, 2,…, s, k = j+1, and when $k > s$, then k =1.

From Equation (3-2), we can know that the inequality of the weight restriction of the "Interval Estimation of Pairwise Weight Ratio" of the mEWM_AR Model does have the characteristic of a non-negative weight lower limit. At the same time, the inequality of the weight restriction of the "Interval Estimation of Pairwise Weight Ratio" of the input and output of (3-2) can be converted into a linear format, as shown in the following formula (3-3):

$$(\frac{\bar{x}_h}{\bar{x}_i+3S_{\bar{x}_i}})v_i - v_h \le 0,\ (\frac{\bar{x}_h+3S_{\bar{x}_h}}{\bar{x}_i})v_i - v_h \ge 0;$$

$$(\frac{\bar{y}_k}{\bar{y}_r+3S_{\bar{y}_r}})u_r - u_k \le 0,\ (\frac{\bar{y}_k+3S_{\bar{y}_k}}{\bar{y}_r})u_r - u_k \ge 0$$

(3-3)

Case Proof and Discussion

In the second section of this article, two literature cases have proved the unreasonable problems of the traditional DEA model, including: (1) Generally has an unreasonable problem of "variable weight is 0"; (2) Because there are differences in the calculation units between variables, it is naturally easy to produce the lower limit of the negative value of the "Interval Estimation of Pairwise Weight Ratio" without the normalization conversion.

Therefore, the two literature cases mentioned in the second section of this article will once again be applied to the mEWM_AR Model research method (two implementation steps) to confirm that our proposed the mEWM_AR Model can indeed be solved at the same time. These two unreasonable problems mentioned above.

As for the implementation method of the example verification of the mEWM_AR Model, it is divided into two

Table 4.1-*a* Weight values of each variable obtained by CW: (v_i, u_r)

v_1	v_2	u_1	u_2	u_3	u_4
0.05429265	0	0.01373395	0	0.007834895	0.04137792

Table 4.1-*b* Weight values of each variable obtained by CCR: (v_i, u_r)

DMU	v_1	v_2	u_1	u_2	u_3	u_4
1	1.023123	0	0.3653841	0	0	0.7891425
2	0.6454844	1.180835	0.4542988	0	0	0.7375602
3	0.9163281	0.3473498	0.2681144	0	0.2099389	0.7301552
4	0.5781360	1.728975	0.5748480	0	0	0.7451465
5	1.039069	0	0	0.9508520	0	0.0649418
6	0.6580889	1.428166	0.4902558	0.0243738	0	0.7861078
7	0.3742723	3.913253	0	0	0	1.328551
8	0.6537489	1.195954	0.4601154	0	0	0.7470036
9	1.059771	0	0	0	0.4013440	0.7499737
10	0	2.862869	1	0	0	0
11	1.022077	0	0	0.8502789	0	0.1688215
12	1.031034	0	0.3773885	0	0.7991161	0
13	1.057977	0	0.0479386	0	0	1.034008
14	1.009897	0	0	0.8401463	0	0.1668097
15	1.004419	0	0.0455118	0	0	0.9816636
16	1	0	0	0	0	1
17	0.9682008	0.3335452	0	0	0	1.073768
18	1.046682	0	0.3737978	0	0	0.8073141
19	1.020825	0	0.0236950	0.7866614	0	0.2210683

Table 4.1-*c* Upper and lower limits of the "Interval Estimation of Pairwise Weight Ratio" in the *m*EWM_AR Model

0.02945	≤	v2/v1	≤	0.39344
2.54167	≤	v1/v2	≤	33.95664
0.21876	≤	u2/u1	≤	2.98021
0.31828	≤	u3/u2	≤	4.18614
0.63032	≤	u4/u3	≤	8.19199
0.12869	≤	u1/u4	≤	1.73246

Table 4.1-*d* Weight values of each variable obtained by the *m*EWM_AR Model

v_1	v_2	u_1	u_2	u_3	u_4
0.0537646	0.0015834	0.0135763	0.0029700	0.0049652	0.0406749

Table 4.2-*a* Weight values of each variable obtained by CW: (v_i, u_r)

v_1	v_2	v_3	u_1	u_2
0.1233569	0	0.0392593	0.1213758	0.0416403

Table 4.2-*b* Weight values of each variable obtained by CCR: (v_i, u_r)

DMU	v_1	v_2	v_3	u_1	u_2
1	3.397685	0	1.081340	3.343118	1.146921
2	4.434590	0	0	0	2.267949
3	0.4211884	0.3557548	0.2603716	0.9167645	0
4	1.432869	0	0.4560218	1.409857	0.4836787
5	4.311058	0	1.423624	4.505103	1.132757
6	0.6051216	0	1.079430	1.257475	0
7	4.898005	0.4760028	0	2.610273	2.003384
8	1.825293	0.1773875	0	0.9727459	0.7465822
9	2.032035	1.509859	0	1.366357	1.803259
10	2.912897	2.164364	0	1.958657	2.584950
11	0.5647010	0.8922196	0	0	1.015332
12	1.315399	1.945120	0.7707414	2.313132	1.782896
13	1.836982	1.551598	1.135592	3.998401	0
14	0.4663309	0.3938842	0.2882780	1.015022	0

Table 4.2-*c* Upper and lower limits of the "Interval Estimation of Pairwise Weight Ratio" in the *m*EWM_AR Model

0.62021	≤	v2/v1	≤	4.03150
0.45595	≤	v3/v2	≤	3.49628
0.18692	≤	v1/v3	≤	1.34222
0.43568	≤	u2/u1	≤	2.73710
0.36535	≤	u1/u2	≤	2.29526

Table 4.2-*d* Weight values of each variable obtained by the *m*EWM_AR Model:

v_1	v_2	u_1	u_2	u_3
0.0448281	0.0712021	0.0333984	0.0804824	0.0704180

stages, namely, Implementation Method and Discussion of Implementation Results.

- Implementation Method

The "Implementation Method" is the two steps of the research method using the *m*EWM_AR Model:

Step 1: First, perform a linear "Normalization" to convert into a dimensionless scoring data table. And find the Mean and Standard Deviation.Step 2: Apply the "Mean and Standard Deviation" to generate the inequality of weight restriction of the Interval Estimation of Pairwise Weight Ratio.

At the same time, the two DMU data tables that have been "Normalized" will once again be applied with the Common Weight (CW) Method and the DEA-CCR Method to obtain the weight values of each variable (v_i, u_r); see "Table 4.1-*a* & Table 4.1-*b*" and "Table 4.2-*a* & Table 4.2-*b*" for details. Then, the research method of the *m*EWM_AR Model proposed in this study is applied again to obtain the upper and lower limits of the "Interval Estimation of Pairwise Weight Ratio"; see "Table 4.1-*d* & Table 4.2-*d*" for details.

The Literature 1 where "Normalization" has been performed: (Mazinani, 2011)

The Literature 2 where "Normalization" has been performed: (Liu *et al.,* 2017)

- Discussion of Implementation Results

After looking at the results of the above 8 tables (Table 4.1-a ~ Table 4.2-d), and comparing the 8 tables in the second section (Table 2.1-a ~ Table 2.2-d), two things can be understood, including: (1) DEA traditional model generally has irrational problem of "variable weight is 0," this matter has nothing to do with Normalization.

(2) The *m*EWM_AR Model proposed in this article provides a Normalization function, the purpose of which is to convert the original DMU data table into a dimensionless scoring data table; therefore, the unit difference of different measures between variables does not exist. At the same time, the theoretical method of "the Importance of the Correlation Between the Variables" of the Expert Weight Method is incorporated into the *m*EWM_AR Model. In the end, the *m*EWM_AR Model will be able to generate a new inequality of the weight restriction for the Interval Estimation of Pairwise Weight Ratio.

That is, the *m*EWM_AR Model proposed in this article has two advantages, including: (1) It can indeed solve the unreasonable problem that the traditional DEA model generally has "variable weight is 0"; (2) It can even let the "Interval Estimation of Pairwise Weight Ratio" have the characteristic of a non-negative lower weight restriction.

Conclusion

The research method of the *m*EWM_AR Model proposed in this article is a combination of theoretical method of Expert Weight Method (EWM) and the concept of AR model. The important point is that the *m*EWM_AR Model still continues the essence of EWM, that is, the theoretical method that emphasizes the connotation of the Importance of the Correlation Between the Variables.

At the same time, the research method of the *m*EWM_AR Model is divided into two stages. The first stage: perform a Linear Normalization on the original DMU score data set to convert it into a dimensionless score data table (Results, see Appendix B). The second stage: Applying the "Mean and Standard Deviation" to generate the inequality of the weight restriction of the Interval Estimation of Pairwise Weight Ratio.

Comparing the research results of "8 tables in Section 2 (Table 2.1-a ~ Table 2.2-d)" and "8 tables in Section 4 (Table 4.1-a ~ Table 4.2-d)," there are two main results produced by the *m*EWM_AR Model, including: (1) It can indeed solve the unreasonable problem that the traditional model of DEA generally has a "variable weight is 0"; (2) In terms of the weight restriction, because it provides The "Normalization" of linear convergence, and the integration of "the Importance of the Correlation Between the Variables" theory method of the Expert Weight Method (EWM), so the *m*EWM_AR Model can indeed make the "Interval Estimation of Pairwise Weight Ratio" have the characteristic of a non-negative lower limit.

This means that the *m*EWM_AR Model will provide a more objective and a more unbiased efficiency evaluation model for decision makers.

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Appendix A

Table A-1 literature 1 (Mazinani *et al*., 2011); (Table 1: *Inputs and outputs of FLPs*)

DMU	*Inputs* (x_1)	*Inputs* (x_2)	*Output* (y_1)	*Output* (y_2)	*Output* (y_3)	*Output* (y_4)
1	20309.56	6405.00	0.4697	0.0113	0.0410	30.89
2	20411.22	5393.00	0.4380	0.0337	0.0484	31.34
3	20280.28	5294.00	0.4392	0.0308	0.0653	30.26
4	20053.20	4450.00	0.3776	0.0245	0.0638	28.03
5	19998.75	4370.00	0.3526	0.0856	0.0484	25.43
6	20193.68	4393.00	0.3674	0.0717	0.0361	29.11
7	19779.73	2862.00	0.2854	0.0245	0.0846	25.29
8	19831.00	5473.00	0.4398	0.0113	0.0125	24.80
9	19608.43	5161.00	0.2868	0.0674	0.0724	24.45
10	20038.10	6078.00	0.6624	0.0856	0.0653	26.45
11	20330.68	4516.00	0.3437	0.0856	0.0638	29.46
12	20155.09	3702.00	0.3526	0.0856	0.0846	28.07
13	19641.86	5726.00	0.2690	0.0337	0.0361	24.58
14	20575.67	4639.00	0.3441	0.0856	0.0638	32.20
15	20687.50	5646.00	0.4326	0.0337	0.0452	33.21
16	20779.75	5507.00	0.3312	0.0856	0.0653	33.60
17	19853.38	3912.00	0.2847	0.0245	0.0638	31.29
18	19853.38	5974.00	0.4398	0.0337	0.0179	25.12
19	20355.00	17402.00	0.4421	0.0856	0.0217	30.02

Table A-2 literature 2 (Liu *et al*., 2017); (Table 9: *Data for 14 passenger airlines values*)

DMU	*Inputs* x_1	*Inputs* x_2	*Inputs* x_3	*Output* y_1	*Output* y_2
1	5,273	3,239	2,003	26,677	697
2	5,895	4,225	4,557	3,081	539
3	24,099	9,560	6,267	124,055	1,266
4	13,565	7,499	3,213	64,734	1,563
5	5,183	1,880	783	23,604	513
6	19,080	8,032	3,272	95,011	572
7	4,603	3,457	2,360	22,112	969
8	12,097	6,779	6,474	52,363	2,001
9	6,587	3,341	3,581	26,504	1,297
10	5,654	1,878	1,916	19,277	972
11	12,599	8,098	3,310	41,925	3,398
12	5,728	2,481	2,254	27,754	982
13	4,715	1,792	2,485	31,332	543
14	22,793	9,874	4,145	122,528	1,404

Appendix B

Two literature cases that have performed Normalization:

Table B-1 A dimensionless score data table for literature 1 (Mazinani *et al*., 2011)

DMU	*Inputs* (x_1)	*Inputs* (x_2)	*Output* (y_1)	*Output* (y_2)	*Output* (y_3)	*Output* (y_4)
1	0.9774	0.3681	0.7091	0.1320	0.4846	0.9193
2	0.9823	0.3099	0.6612	0.3937	0.5721	0.9327
3	0.9760	0.3042	0.6630	0.3598	0.7719	0.9006

4	0.9650	0.2557	0.5700	0.2862	0.7541	0.8342
5	0.9624	0.2511	0.5323	1.0000	0.5721	0.7568
6	0.9718	0.2524	0.5546	0.8376	0.4267	0.8664
7	0.9519	0.1645	0.4309	0.2862	1.0000	0.7527
8	0.9543	0.3145	0.6639	0.1320	0.1478	0.7381
9	0.9436	0.2966	0.4330	0.7874	0.8558	0.7277
10	0.9643	0.3493	1.0000	1.0000	0.7719	0.7872
11	0.9784	0.2595	0.5189	1.0000	0.7541	0.8768
12	0.9699	0.2127	0.5323	1.0000	1.0000	0.8354
13	0.9452	0.3290	0.4061	0.3937	0.4267	0.7315
14	0.9902	0.2666	0.5195	1.0000	0.7541	0.9583
15	0.9956	0.3244	0.6531	0.3937	0.5343	0.9884
16	1.0000	0.3165	0.5000	1.0000	0.7719	1.0000
17	0.9554	0.2248	0.4298	0.2862	0.7541	0.9313
18	0.9554	0.3433	0.6639	0.3937	0.2116	0.7476
19	0.9796	1.0000	0.6674	1.0000	0.2565	0.8935

Table B-2 A dimensionless score data table for literature 2 (Liu *et al*., 2017)

DMU	x_1	x_2	x_3	y_1	y_2
1	0.1997	0.3099	0.2973	0.2126	0.1900
2	0.2255	0.4097	0.6918	0.0224	0.1435
3	0.9809	0.9500	0.9559	0.9975	0.3575
4	0.5438	0.7413	0.4842	0.5193	0.4449
5	0.1960	0.1722	0.1089	0.1878	0.1359
6	0.7726	0.7953	0.4933	0.7634	0.1532
7	0.1719	0.3320	0.3524	0.1758	0.2701
8	0.4829	0.6684	0.9879	0.4196	0.5738
9	0.2542	0.3202	0.5410	0.2112	0.3666
10	0.2155	0.1720	0.2839	0.1529	0.2710
11	0.5037	0.8020	0.4992	0.3355	0.9849
12	0.2186	0.2331	0.3361	0.2212	0.2739
13	0.1766	0.1633	0.3718	0.2501	0.1447
14	0.9267	0.9819	0.6282	0.9852	0.3981